Quiz 2.1: Sample Answers

1. Find the slope of the tangent line to the parabola $f(x) = 3x^2 - x$ at the point $x = -1$, without evaluating the limit.

   The slope of the tangent line at $x = -1$ is given by the limit:

$$\lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1} \frac{[-2x^2 - x] - [-2(-1)^2 - (-1)]}{x + 1}$$

$$= \lim_{x \to -1} \frac{-2x^2 - x + 1}{x + 1}$$

2. Use the expression

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

   to find the slope of the tangent line to $f(x) = -x^2 + 3x$ at $x = 1$, without evaluating the limit.

   Here, $a = 1$, so we have the limit:

$$\lim_{h \to 0} \frac{f(1 + h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{[-(1 + h)^2 + 3(1 + h)] - [-(1)^2 + 3(1)]}{h}$$

$$= \lim_{h \to 0} \frac{-1 - 2h - h^2 + 3 + h - 2}{h}$$

$$= \lim_{h \to 0} \frac{-h^2 + h}{h}$$

3. Use the $h \to 0$ limit to find $f'(-1)$ for $f(x) = \frac{x+1}{2x+1}$, without evaluating the limit.

   We have:

$$\lim_{h \to 0} \frac{f(-1 + h) - f(-1)}{h}$$

$$\lim_{h\to 0} \frac{\frac{(-1+h)+1}{2(-1+h)+1} - \frac{(-1)+1}{2(-1)+1}}{h}$$

$$= \lim_{h\to 0} \left(\frac{h}{-2+2h+1} - 0 \right) \left(\frac{1}{h} \right)$$

$$= \lim_{h\to 0} \frac{1}{2h-1}$$

4. Use the $h \to 0$ limit to find $f'(x)$ for $f(x) = \sqrt{3x-1}$, without evaluating the limit.

   We have:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{3(x+h)-1} - \sqrt{3x-1}}{h}$$

   Multiply and divide by the reciprocal:

$$= \lim_{h\to 0} \frac{\sqrt{3(x+h)-1} - \sqrt{3x-1}}{h} \left(\frac{\sqrt{3(x+h)-1} + \sqrt{3x-1}}{\sqrt{3(x+h)-1} + \sqrt{3x-1}} \right)$$

$$= \lim_{h\to 0} \frac{[3(x+h)-1] - [3x-1]}{h\left(\sqrt{3(x+h)-1} + \sqrt{3x-1}\right)}$$

$$= \lim_{h\to 0} \frac{3h}{h\left(\sqrt{3x+3h-1} + \sqrt{3x-1}\right)}$$

5. Use the $h \to 0$ limit to find $f'(x)$ for $f(x) = \frac{2}{3x^2}$, without evaluating the limit.

   We have:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{3(x+h)^2} - \frac{2}{3x^2}}{h}$$

Find a common denominator:

$$= \lim_{h \to 0} \frac{6x^2 - 6(x+h)^2}{9x^2(x+h)^2} \left(\frac{1}{h}\right)$$

$$= \lim_{h \to 0} \frac{6x^2 - (6x^2 + 12xh + 6h^2)}{9hx^2(x+h)^2}$$

$$= \lim_{h \to 0} \frac{-12xh - 6h^2}{9hx^2(x+h)^2}$$

$$= \lim_{h \to 0} \frac{-4x - 2h}{3x^2(x+h)^2}$$